

Důkazy kombinatorických identit

1.

$$\binom{n}{k} = \binom{n}{n-k}$$
$$\binom{p}{r} \binom{p-r}{q-r} = \binom{p}{q} \binom{q}{r}$$

2.

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

3.

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{k}a^{n-k}b^k + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

4.

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$$

5.

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n \binom{n}{n}$$

6.

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \binom{n}{3}^2 + \dots + \binom{n}{n}^2$$
$$\binom{2n}{n} = \binom{k}{0} \binom{2n-k}{n} + \binom{k}{1} \binom{2n-k}{n-1} + \binom{k}{2} \binom{2n-k}{n-2} + \dots + \binom{k}{k} \binom{2n-k}{0}$$
$$\binom{m+p}{m} = \binom{p}{0} \binom{m}{m} + \binom{p}{1} \binom{m}{m-1} + \binom{p}{2} \binom{m}{m-2} + \dots + \binom{p}{m} \binom{m}{0}$$

7.

$$\binom{p}{q} = \binom{p-1}{q-1} + \binom{p-2}{q-1} + \binom{p-3}{q-1} + \dots + \binom{q}{q-1} + \binom{q-1}{q-1}$$

8.

$$n \cdot 2^{n-1} = 1 \binom{n}{1} + 2 \binom{n}{2} + 3 \binom{n}{3} + \dots + (n-1) \binom{n}{n-1} + n \binom{n}{n}$$